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Description automatically generated

The Euclidean norm of a vector v = [v1, v2, ..., vn] is defined as the square root of the sum of the squares of its components:

||v|| = sqrt(v1^2 + v2^2 + ... + vn^2)

To compute the Euclidean norm for each of the given vectors:

[1, 0, 2]:

||[1, 0, 2]|| = sqrt(1^2 + 0^2 + 2^2) = sqrt(5)

[3, 4]:

||[3, 4]|| = sqrt(3^2 + 4^2) = 5

[-7, 2, -4, sqrt(12)]:

||[-7, 2, -4, sqrt(12)]|| = sqrt((-7)^2 + 2^2 + (-4)^2 + sqrt(12)^2) = sqrt(99)

Text

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The corresponding unit length vector of a non-zero vector v is obtained by dividing v by its Euclidean norm, i.e., v divided by ||v||.

Let's compute the corresponding unit length vector for each of the given vectors:

[3, 4]:

The Euclidean norm of [3, 4] is ||[3, 4]|| = sqrt(3^2 + 4^2) = 5.

So, the corresponding unit length vector is [3/5, 4/5].

[-1, -2, 3]:

The Euclidean norm of [-1, -2, 3] is ||[-1, -2, 3]|| = sqrt((-1)^2 + (-2)^2 + 3^2) = sqrt(14).

So, the corresponding unit length vector is [-1/sqrt(14), -2/sqrt(14), 3/sqrt(14)].

[-7, 2, -4, sqrt(12)]:

The Euclidean norm of [-7, 2, -4, sqrt(12)] is ||[-7, 2, -4, sqrt(12)]|| = sqrt((-7)^2 + 2^2 + (-4)^2 + sqrt(12)^2) = sqrt(99).

So, the corresponding unit length vector is [-7/sqrt(99), 2/sqrt(99), -4/sqrt(99), sqrt(12)/sqrt(99)].

Text

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The inner product of two vectors u = [u1, u2, ..., un] and v = [v1, v2, ..., vn] is defined as:

u·v = u1v1 + u2v2 + ... + un\*vn

To compute the inner product between the given vectors [3, -2, 2] and [1, 2, 2]:

[3, -2, 2] · [1, 2, 2] = 3\*1 + (-2)2 + 22 = 3 - 4 + 4 = 3

To compute the angle between two vectors u and v, we can use the formula:

cos(theta) = (u·v) / (||u|| \* ||v||)

where ||u|| and ||v|| are the Euclidean norms of u and v, respectively.

So, for the given vectors [3, -2, 2] and [1, 2, 2], we have:

||[3, -2, 2]|| = sqrt(3^2 + (-2)^2 + 2^2) = sqrt(17)

||[1, 2, 2]|| = sqrt(1^2 + 2^2 + 2^2) = 3

Therefore, the angle between the two vectors is:

cos(theta) = (3) / (sqrt(17) \* 3) ≈ 0.5518

theta = arccos(0.5518) ≈ 0.983 radians ≈ 56.24 degrees

So, the angle between [3, -2, 2] and [1, 2, 2] is approximately 56.24 degrees.

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Projection is a way to represent a vector as a combination of vectors in a subspace. Specifically, the projection of a vector u onto a subspace spanned by a set of vectors {v1, v2, ..., vn} is given by:

proj\_u(S) = ((u · v1) / ||v1||^2) \* v1 + ((u · v2) / ||v2||^2) \* v2 + ... + ((u · vn) / ||vn||^2) \* vn

where "·" represents the dot product, "||v||" represents the Euclidean norm of v, and S is the subspace spanned by {v1, v2, ..., vn}.

To answer the questions:

The subspace spanned by [1, 1] is the line passing through the origin and [1, 1]. To project [5, 2] onto this subspace, we can use the formula:

proj\_u(S) = ((u · v) / ||v||^2) \* v

where u = [5, 2], v = [1, 1].

proj\_u(S) = (([5, 2] · [1, 1]) / ||[1, 1]||^2) \* [1, 1]

= ((5 + 2) / 2) \* [1, 1]

= 3 \* [1, 1]

= [3, 3]

Therefore, the projection of [5, 2] onto the subspace spanned by [1, 1] is [3, 3].

The subspace spanned by [1, -1, -1] is a line passing through the origin and [1, -1, -1]. To project [0, 2, 1] onto this subspace, we can use the formula:

proj\_u(S) = ((u · v) / ||v||^2) \* v

where u = [0, 2, 1], v = [1, -1, -1].

proj\_u(S) = (([0, 2, 1] · [1, -1, -1]) / ||[1, -1, -1]||^2) \* [1, -1, -1]

= ((0 - 2 - 1) / 3) \* [1, -1, -1]

= (-1/3) \* [1, -1, -1]

= [-1/3, 1/3, 1/3]

Therefore, the projection of [0, 2, 1] onto the subspace spanned by [1, -1, -1] is [-1/3, 1/3, 1/3].

Project [5, 2] onto the subspace spanned by vectors [2, 3], [1, 1]:

To find the projection, we first need to find the matrix A whose columns are the basis vectors for the subspace:

A = [2 1; 3 1]

To apply the formula, we also need to compute (A^T A)^-1 A^T:

(A^T A)^-1 A^T = ([2 3; 1 1][2 1; 3 1])^-1 [2 3; 1 1]^T = (13/14)[2 1; 3 1]^T

Finally, we can compute the projection:

projV(x) = A(A^T A)^-1 A^T x = 2 1; 3 1[2 1; 3 1]^T [5; 2] = [3; 2]

Therefore, the projection of [5, 2] onto the subspace spanned by [2, 3], [1, 1] is [3, 2].

What is the projection of [1,−1, 1] onto the subspace spanned by vectors [0, 0,−1], [2, 0, 1]?

To find the projection, we first need to find the matrix A whose columns are the basis vectors for the subspace:

A = [0 2; 0 0; -1 1]

To apply the formula, we also need to compute (A^T A)^-1 A^T:

(A^T A)^-1 A^T = ([0 0 -1; 2 0 1][0 2; 0 0; -1 1])^-1 [0 0 -1; 2 0 1]^T = (1/5)[0 0 -1; 0 0 2]^T

Finally, we can compute the projection:

projV(x) = A(A^T A)^-1 A^T x = 0 2; 0 0; -1 1[0 0 -1; 0 0 2]^T [1; -1; 1] = [-1/5; 2/5; 2/5]

Therefore, the projection of [1,−1, 1] onto the subspace spanned by [0, 0,−1], [2, 0, 1] is [-1/5, 2/5, 2/5].

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